

PAPER EVALUATION REPORT

AIR Journals — AI-Powered Academic Review

(Under Strict Scholarly Supervision)

Submission ID	AIR-2026-000962
Paper Title	Exactly Three Single-Amplification Cycles in GCD-Augmented Fibonacci Recurrences
Author	Mosab Hawarey
Submission Date	07 October 2026
Journal	AIR Journal of Mathematics and Computational Sciences (DOI: 10.65737/AIRMCS) (ISSN: 3142-7197)
Review Completed	09 October 2026
Reviewer	AIR Journals AI Review System
Report Version	Final Evaluation Report

Page and line references refer to the line-numbered manuscript as submitted; pagination may differ slightly between viewers.

EXECUTIVE SUMMARY

Overall Score: 83/100

Editorial Decision: Minor Revisions

Decision Rationale: This is a mathematically sound and fully reproducible paper. It proves a complete classification of single-amplification cycles and discovers a new 4-cycle that corrects a published claim. We verified every proof and reproduced every numerical result independently. Minor revisions are needed:

- report the extended census completely;
- situate the work within the closely related literature on cycle classification for Collatz-type maps;
- correct one cross-reference;
- make two small citation corrections.

Primary Strengths:

- **Complete, correct proofs.** The classification (Theorem 10) follows from a clean chain of results (Lemmas 1–9, Theorem 7), all independently verified. This includes an exhaustive search over all coprime states with entries up to 2,500, which finds exactly the three cycles of the theorem.
- **A concrete new discovery.** The 4-cycle $(29, 47) \rightarrow (47, 77) \rightarrow (77, 125) \rightarrow (125, 203)$, with multiplier 7, corrects an earlier published claim and explains the previously unexplained seeds $p = 9, 15$ and 27 .
- **Exemplary reproducibility.** The public script reproduces every number in the paper and regenerates Figure 1 exactly.

Primary Concerns:

- **Incomplete census report.** The census for seeds up to 200 is reported only in part (p. 8, ll. 318–319). The full results include a further multi-amplification cycle and a higher share of seeds with no detected cycle.
- **Missing closest precedent.** The paper does not engage with the classical literature on classifying cycles of Collatz-type maps by the number of events per period, which is the closest methodological precedent (p. 2, ll. 62–67).

- **Narrow significance argument.** The significance of the problem is argued mainly in terms of the author's own earlier papers. In addition, one cross-reference (p. 7, l. 277) overstates what a single part of Corollary 11 proves.

Timeline for Resubmission: Less than 1 week (if AI-assisted); 2–4 weeks otherwise

COMPREHENSIVE EVALUATION RESULTS

Citation Integrity Verification (Step 1)

Status: **PASS**

Verification Level: 100% — Every citation checked

Citation Statistics:

- Total bibliography entries: 13
- Bibliography entries cited in text: 13 (100%)
- Reference numbers cited in text that have bibliography entries: 13 of 13 (100%)
- References confirmed as genuine publications against publisher, journal and database records: 13 of 13, with no errors in bibliographic details

Issues Identified:

Uncited Bibliography Entries (Padding): Count: 0 (0%) — Severity: None

Missing Bibliography Entries: Count: 0 — Severity: None

Additional Citation Issues:

- **Formatting consistency:** Good. Numbered citations with APA-style entries, used consistently. There are minor completeness items (listed below).
- **Self-citation rate:** 15.4% (2 of 13). Moderate but justified, since the paper extends and corrects the author's earlier work [2], [3]. However, the text does not state explicitly that these are the author's own papers.
- **Figure/table attribution:** Complete. All figures and tables are original and generated by the author's cited code.
- **Data source citations:** One minor gap. The “published values for $p \leq 49$ ” (p. 7, l. 286) are not attributed to a source.

Required Citation Corrections:

Critical (Must Fix): None

Important (Should Fix):

1. p. 7, l. 286: cite the source of the “published values for $p \leq 49$ ” (presumably [2] and/or [3]).
2. p. 1, ll. 34 and 39, and Abstract, ll. 19–20: identify [2] and [3] explicitly as the author's own earlier work (e.g., “the author's earlier work [2]”).

Minor (Recommended):

1. Add stable URLs for [9], [11], [12] and [13].
2. Harmonize the format of [2] and [3] with the article-number style of [11]–[13].
3. Add a retrieval date to the OEIS entry [1].
4. Give a page or identity number in [8] for identities (I1)–(I4) (p. 2, l. 88).
5. Archive the reproduction code with a persistent identifier (DOI) and cite it with its version.

Citation Integrity Conclusion: Citation integrity is excellent. Every bibliography entry is cited, every citation has an entry, and every reference is genuine. The two important corrections are small improvements in attribution and transparency.

Plagiarism Content Assessment (Step 2)

Status: **PASS**

Tool Used: Premium Professional Service

Similarity Scores:

- Raw similarity score: 5%
- Adjusted similarity score (after removing legitimate matches): 0%
- Assessment: Green (0–14%)

Analysis: All 16 matches were classified as benign:

- 13 matches: bibliographic reference entries, i.e. titles and journal names of properly cited works, including the publisher pages of [2] and [3]
- 2 matches: boilerplate declarations (the AI-assistance disclosure wording, and the conflict-of-interest and funding statement)
- 1 match: a common 12-word definitional phrase in Definition 1 (“the multiplier of the cycle is the product of its ... step multipliers”)

Apart from that phrase, there are no matches anywhere in the main text, including the results sections (Sections 4–5).

Self-Plagiarism Check:

- Author's prior work matched: Yes, but only the bibliographic metadata of [2] and [3]
- Overlap: about 0% of the main text
- Properly disclosed: Partially. The papers are described as “earlier work” and “the sequel”, but not explicitly as the author's own.
- Properly self-cited: Yes, precisely and throughout
- Assessment: Acceptable

Required Corrections: None (the self-citation wording is covered under the citation corrections above)

Plagiarism Conclusion: No plagiarism detected. The text is original, including where the paper builds closely on the author's earlier papers.

Quality Evaluation Results (Step 3)

Overall Quality Score: **83/100**

Score Breakdown by Category:

CATEGORY A: CONCEPTUAL FOUNDATION

Score: 22/30

Evaluation:

The paper addresses a precise, verifiable gap. The companion paper [3] left higher-period orbits open, while asserting in its Corollary 5 that no further coprime periodic orbits exist (p. 1, ll. 39–42). The present paper settles the single-amplification case completely and refutes that claim with an explicit cycle (pp. 1–2, ll. 44–48). It also explains the anomalous seeds of [2] (p. 2, ll. 50–51). The wider significance is less developed. The motivation rests largely on gaps in the author's own two recent papers. The connections to the Greene–Niedzielski framework and to Collatz-type maps get one sentence each (p. 2, ll. 62–65), without explaining what the result teaches about nonlinear integer recurrences more generally.

Literature use is accurate and purposeful. The link to the shifted-Fibonacci gcd literature [11–13] is genuine synthesis: the “classical unbounded case” is shown to be exactly what eliminates the odd periods (p. 6, ll. 231–232). However, the paper does not engage the most

relevant body of prior work. Classifying the cycles of a map by the number of “events” per period is a classical theme in $3x+1$ theory. It runs from Böhm and Sontacchi's cycle equation (1978) and Steiner's theorem on circuits (1978) to Simons and de Weger's m -cycles (2005). The distinction here between single and multiple amplifications is a direct analogue. Acknowledging it would sharpen both the novelty claim and the open problem. The exposition is precise. All the key notions are defined before use: state, multiplier, amplification, single-amplification cycle, normalized state and shifted state. Scope labels S1–S3 delimit the claims, and the notation is carefully reconciled with [3] (p. 3, ll. 95–96). A few minor issues remain:

- “Geometric lock-in” (p. 4, l. 148) is used via [3] without a local definition.
- The Abstract (ll. 15–16) uses “fixed point” for the solution of the linear system, although the same term also names the cycle (1, 2).
- The index convention in Lemma 3(a)(i) becomes clear only from its proof.

Key Strengths:

- A precise, falsifiable gap, closed by an explicit counterexample to a published claim
- Rigorous definitions and scope labels that make every claim easy to delimit

Areas for Improvement:

- Add a paragraph (after p. 2, l. 67) explaining why this recurrence is a valuable model case beyond the author's own earlier papers
- Situate the work against the Collatz cycle-classification literature (Böhm–Sontacchi; Steiner; Simons–de Weger) in the Introduction and Discussion
- Define “geometric lock-in” locally (p. 4, l. 148)
- In the Abstract, write “unique solution of the resulting linear system” instead of “fixed point”
- Add one sentence explaining the index convention of Lemma 3(a)(i)

CATEGORY B: METHODOLOGICAL RIGOR

Score: 27/30

Evaluation:

The method fits the question exactly. The reduction runs as follows:

- Shifting the cofactors by one linearizes every non-amplifying step (Lemma 1). This reduces single-amplification cycles to a 2×2 cycle equation (Lemma 3).
- The Fibonacci norm form then forces the multiplier to be L_q or $L_q + 1$ (Lemma 5, Theorem 7).
- Fibonacci–Lucas gcd identities eliminate every period except 1, 2 and 4 (Lemmas 8–9).

Each step is necessary, and together they give a complete classification. Computation is used appropriately, for the census and for finite checks, and never in place of proof.

Execution is near-flawless. We verified every proof line by line, including:

- the adjugate in Proposition 4
- the norm identity
- the sign analysis of D
- the congruence argument in Theorem 7
- the sign bookkeeping in Lemma 8
- the four identities in Lemma 9
- the inequality in Corollary 11(b)

We also confirmed the results computationally. An exhaustive search over all coprime states with entries up to 2,500 finds exactly the three cycles of Theorem 10. The single inaccuracy is the cross-reference at p. 7, l. 277: uniqueness of the period-2 orbit ([3, Theorem 8]) needs Corollary 11(b) as well as part (a).

Reproducibility is exemplary. The public script runs in under a minute, reproduces every numerical value in the paper, and regenerates Figure 1 identically. The remaining gaps are minor:

- The code has no persistent archive.
- One output block is labelled “Remark 6”, which does not exist in the manuscript.
- For the larger census, the script reports only an aggregate “other cycle” category.

Key Strengths:

- An elegant, complete reduction (linearization, norm form, gcd exclusion) matched exactly to the question
- Correct, fully explicit proofs and fully reproducible computations

Areas for Improvement:

- p. 7, l. 277: change “Part (a)” to “Parts (a) and (b)”
- Archive the code with a DOI linked to a tagged release
- Align the script's labels with the manuscript
- Have the script print the full census for seeds up to 200
- Briefly discuss whether the linearization approach could extend to cycles with two amplifications (Section 6)

CATEGORY C: EVIDENCE & REASONING

Score: 18/20

Evaluation:

Every theorem has a complete proof, and every numerical claim is exact and reproducible. This includes the corrected values attributed to errors in [2] and [3] (p. 7, ll. 306–308) and the five counts of Table 2. The argument runs as a clean deductive chain from Lemma 1 to Theorem 10. The Abstract, Table 1, Discussion and Conclusions are mutually consistent. The paper itself shows where its main theorem stops, by exhibiting a period-22 cycle with six amplifications (p. 8, ll. 323–325).

One empirical statement is reported selectively. For seeds with $1 \leq a, b \leq 200$, the paper gives only the 4-cycle (38.4%) and 2-cycle (32.4%) shares (p. 8, ll. 318–319). Our full recomputation (24,463 seeds) gives:

- 4-cycle: 38.4%
- 2-cycle: 32.4%
- no repeated state within 1,200 steps: 13.4% (unchanged at 2,400 steps)
- period-22 cycle: 8.5%
- fixed point: 7.4%
- a further cycle of period 65 through (4003, 6477), with 19 amplifications, reached by the seeds (3, 187) and (187, 191)

The paper mentions neither the rising share of undetected seeds nor the period-65 cycle. Neither affects the theorems, but both bear directly on the discussion of how common the cycles are and on scope items S1 and S3. The one overstated inference (p. 7, l. 277) is noted above.

Key Strengths:

- Complete proofs for every theorem, and exact, reproducible numbers
- A tight deductive structure whose limits are demonstrated by an explicit counterexample

Areas for Improvement:

- Report the full census for seeds up to 200, including the period-65 cycle as a second multi-amplification example (p. 8, ll. 318–325; p. 9, l. 338)
- Add one sentence on orbit growth for seeds with no detected cycle, to inform the open question at p. 9, l. 345. For example, for seed (1, 45) the cofactors reach about 211 digits after 2,400 steps.

*CATEGORY D: TRANSPARENCY AND INTEGRITY***Score:** 9/10**Evaluation:**

Transparency is a clear strength:

- Scope labels S1–S3 (p. 3, ll. 98–102) map one-to-one onto the limitations list (p. 9, ll. 336–343).
- Hypothesis S1 is shown to be essential by an explicit counterexample.
- The census caveats (step limit, undetected seeds, dependence on box size) are stated.
- The author openly corrects his own earlier published claims and values (p. 1, l. 41; p. 6, l. 263; p. 7, ll. 306–308).

The assumptions are fully explicit:

- The coprimality assumption A1 is justified by [2, Theorem 4] (p. 2, ll. 74–77).
- The conventions for negative indices are stated, and every lemma states its hypotheses.
- The paper notes exactly what Theorem 7 does and does not use (p. 5, l. 197).
- The robustness of the census is tested by doubling the step limit and enlarging the box.

The one gap is that the caveats from the larger census (the higher undetected share and the additional cycle) are not reported.

Key Strengths:

- Honest, specific limitations tied to explicit scope labels, including candid correction of the author's own earlier errors
- Exemplary statement of assumptions and of what each result depends on

Areas for Improvement:

- Extend S1 and S3 in Section 6 with the findings of the larger census (13.4% undetected; the period-65 cycle)

*CATEGORY E: CONTRIBUTION AND IMPACT***Score:** 7/10**Evaluation:**

The paper's novelty is clear:

- a previously unreported 4-cycle, which we confirmed independently and for which we found no earlier report
- a complete classification of single-amplification cycles

- a purely algebraic proof that replaces a partly computational argument
- an explanation of the anomalous seeds of [2]

The paper extends the author's own research programme rather than founding a new one. Its central technique, reducing cycles with a fixed event pattern to a cycle equation, has recognizable antecedents in $3x+1$ theory, although the Fibonacci-matrix linearization and the norm-form constraint are new here.

Within its area the paper is definitive. The single-amplification case is closed, a published claim is corrected, and a well-posed open problem (cycles with several amplifications) is supplied with data. Its broader reach is modest, because the recurrence has so far been studied mainly in the author's own papers and the paper does not discuss whether the method transfers to other recurrences.

Key Strengths:

- A concrete new discovery, proved and computationally confirmed, that corrects the literature

Areas for Improvement:

- State the novelty relative to the Collatz cycle-equation literature
- Add 2–3 sentences in Section 6 on which other recurrences admit the same linearization
- Frame the multi-amplification problem as the analogue of the m-cycle problem

Summary of Score Breakdown by Category:

Category	Score	Maximum
Conceptual Foundation	22	30
Methodological Rigor	27	30
Evidence & Reasoning	18	20
Transparency and Integrity	9	10
Contribution and Impact	7	10
TOTAL	83	100

EDITORIAL DECISION

Decision: MINOR REVISIONS

Score Band: **83/100**

Decision Thresholds	Range
Accept	90–100
Minor Revisions	80–89 ← This paper
Major Revisions	70–79
Major Revisions (Uncertain)	60–69
Reject (May Resubmit)	50–59
Reject	0–49

Factors Considered in Decision:

- Overall quality score: 83/100
- Citation integrity status: PASS (no impact)
- Plagiarism assessment: PASS (no impact)
- Pattern: very strong rigor, evidence and transparency; weaker framing, literature positioning and completeness of reporting

- Feasibility: all required changes are text additions or corrections; no new mathematics is needed
- Significance: a definitive, correct result in a specialized area

Detailed Justification

This paper scores 83/100, three points above the Minor Revisions threshold. The mathematics is correct and complete: every proof was verified and every numerical result was reproduced independently. The paper's main limitations lie in how the work is positioned and reported, not in its rigor.

The revisions required are specific and additive:

- report the census for seeds up to 200 in full;
- situate the work within the literature on cycle classification for Collatz-type maps;
- correct one cross-reference;
- make two small citation corrections.

None of these requires new mathematics or new data; the author's own code has already computed the extended census. The theorems, proofs and conclusions will remain unchanged.

Both integrity checks passed without concerns. Citation integrity is excellent: all 13 references are cited, genuine and accurately described. The adjusted similarity is 0%. This gives high confidence that the score reflects the paper's scholarly merit.

The contribution justifies publication: a complete, correct classification theorem, a new cycle that corrects a published claim, and an explanation of a previously open phenomenon. Its reach is currently specialized, and the recommended additions on related literature and transferability would broaden its relevance. We expect the revised paper to meet the acceptance threshold.

Integration of All Evaluation Steps:

- **Step 1 (citation integrity):** a clean, fully verified bibliography, with two small attribution items.
- **Step 2 (plagiarism):** fully original text.
- **Step 3 (quality evaluation):** very strong rigor, evidence and transparency. Improvements are needed in literature positioning, the framing of significance, and the completeness of the extended census.

Together, these support a Minor Revisions decision with high confidence of acceptance after revision.

REVISION REQUIREMENTS

Critical Issues (Must Address for Acceptance)

Priority 1 — Essential:

1. Complete the Extended Census Report

- **Current Problem:** For seeds with $1 \leq a, b \leq 200$, only the 4-cycle (38.4%) and 2-cycle (32.4%) shares are reported (p. 8, ll. 318–319). The same computation also shows:
 - 13.4% of seeds with no repeated state (versus 10.0% for the smaller box)
 - 8.5% reaching the period-22 cycle
 - 7.4% reaching the fixed point

- a further cycle of period 65 through (4003, 6477), with 19 amplifications and cycle multiplier $3^{15} \cdot 5^3 \cdot 7^2 \cdot 19 \cdot 23$, reached by the seeds (3, 187) and (187, 191)
 - **Required Fix:**
 - Report the full distribution, either as a second column of Table 2 or in the text.
 - Add the period-65 cycle under S1 as a second multi-amplification example (p. 8, l. 323ff; p. 9, l. 338).
 - Note under S3 (p. 9, ll. 342–343) that the share of seeds with no detected cycle grows with the box.
 - **Success Criterion:** All categories and the period-65 cycle are reported, with values matching an independent recomputation.
 - **Importance:** The extended census is presented as a robustness check. Reporting it in full supports the discussion of how common the cycles are and states scope items S1 and S3 accurately.
2. **Situate the Work in the Literature on Cycle Classification**
- **Current Problem:** Classifying cycles by the number of “events” per period is a classical theme for Collatz-type maps, but the paper does not engage this literature. [7] is cited only in general terms (p. 2, l. 65; p. 9, ll. 345–346).
 - **Required Fix:**
 - Add a short paragraph in the Introduction (p. 2, ll. 62–67) and one or two sentences in the Discussion.
 - Relate single-amplification cycles to Steiner's circuits, and multi-amplification cycles to m-cycles.
 - Cite Böhm & Sontacchi (1978), Steiner (1978) and Simons & de Weger (2005).
 - State what is new in the present approach (the Fibonacci-matrix linearization and the norm-form constraint).
 - **Success Criterion:** The three works are cited accurately, the analogy is explicit, and the novelty claim is stated relative to them.
 - **Importance:** Positions the contribution correctly, sharpens its novelty, and frames the open multi-amplification problem for a wider readership.
3. **Correct the Cross-Reference at p. 7, l. 277**
- **Current Problem:** The text says “Part (a) gives a purely algebraic proof of [3, Theorems 6 and 8]”. Uniqueness of the period-2 orbit also requires part (b), which excludes period-2 cycles with two amplifications.
 - **Required Fix:** Change it to “Parts (a) and (b) give ...”.
 - **Success Criterion:** The sentence attributes the result correctly.
 - **Importance:** The paper's account of its own logic must be accurate.
4. **Make Two Citation Corrections**
- **Current Problem:** The “published values for $p \leq 49$ ” (p. 7, l. 286) have no source, and [2] and [3] are not identified as the author's own work (p. 1, ll. 34, 39; Abstract, ll. 19–20).
 - **Required Fix:** Cite the source at l. 286. Use wording such as “the author's earlier work [2]” and “the author's sequel [3]”.
 - **Success Criterion:** Both changes are visible in the revised text.

- **Importance:** Complete attribution and transparency, especially because the paper corrects these works.

Important Issues (Strongly Recommended)

Priority 2 — Significantly Improve Paper:

1. Strengthen the Significance Argument

- **Current State:** The motivation rests mainly on gaps in the author's own papers (pp. 1–2, ll. 33–67).
- **Recommended Improvement:** Explain why this recurrence is a valuable model case. It is a rare nonlinear integer recurrence whose cycles can be classified exactly; the method may transfer to other conditional-division maps; and the result gives exact growth laws for a large share of seeds.
- **Expected Benefit:** Broader relevance and readership.

2. Clarify Terminology and Notation

- **Current State:**
 - “Geometric lock-in” is not defined locally (p. 4, l. 148).
 - The Abstract says “fixed point of the resulting linear system” (ll. 15–16).
 - The index convention in Lemma 3(a)(i) is implicit.
- **Recommended Improvement:**
 - Add a one-clause definition of geometric lock-in.
 - Write “unique solution of the resulting linear system”.
 - Add one sentence after Lemma 3(a) explaining that condition (i) says steps 0, ..., $q-2$ are non-amplifying.
- **Expected Benefit:** Removes the remaining ambiguities for readers new to [3].

3. Report Evidence on Seeds with No Detected Cycle

- **Current State:** 10.0% of seeds show no repeated state, and the paper gives no information on how they behave (p. 9, ll. 342–345).
- **Recommended Improvement:** Report how the cofactors grow for these orbits (e.g., their size after 1,200 and 2,400 steps).
- **Expected Benefit:** Informs the open question of whether every seed reaches a cycle.

4. Expand the Comparison with Greene–Niedzielski [4]

- **Current State:** One sentence each (p. 2, ll. 62–63; p. 4, ll. 148–152).
- **Recommended Improvement:** Add one or two sentences on what their method yields in their setting and what changes with a gcd divisor.
- **Expected Benefit:** A firmer connection to prior work.

5. Discuss Extensions of the Method

- **Current State:** Not discussed.
- **Recommended Improvement:** Comment on whether the approach can reach cycles with two amplifications, or other recurrences (including the higher-order analogues of S_2).
- **Expected Benefit:** Clearer research directions and broader impact.

Minor Issues (Suggested for Polish)

Priority 3 — Enhancement:

1. Code: archive it with a DOI linked to a tagged release, rename the script's "Remark 6" output block, and make the script print the full census for seeds up to 200.
2. Figure 1 (p. 8, l. 311): note that isolated exact four-step ratios of 7 occur before lock-in ($p = 9$ at $n = 8$ and 23; $p = 27$ at $n = 12$), so that the hollow markers at 7.0 are not misread.
3. Consider calling G the "cycle multiplier" throughout, to distinguish it from step multipliers.
4. References: add URLs for [9] and [11]–[13]; harmonize [2] and [3] with [11]–[13]; add an OEIS retrieval date to [1]; give a pinpoint in [8] for (I1)–(I4).

Revision Timeline and Expectations

Timeline: Less than 1 week (if AI-assisted); 2–4 weeks otherwise

Scope: Small, focused improvements to address specific issues

Re-review: Rapid review of revised sections only

Acceptance Probability: 95%+ if all feedback is addressed

Guidance: The changes are straightforward and additive; the paper's theorems and proofs will not need modification.

COMPREHENSIVE FEEDBACK FOR AUTHORS

What This Paper Does Well

This is a carefully constructed and genuinely satisfying piece of mathematics. Shifting the cofactors by one turns the dynamics between amplifications into the Fibonacci matrix. From there the classification follows through a short chain of exact arguments: the norm form fixes the multiplier as a Lucas number or one more, and classical gcd identities eliminate every period except 1, 2 and 4. The 4-cycle $(29, 47) \rightarrow \dots \rightarrow (125, 203)$, with multiplier 7, is a concrete new result. Proposition 12 uses it to explain the behaviour of the seeds 9, 15 and 27, which was previously left open.

The paper is also exemplary in its transparency:

- Its scope labels delimit every claim.
- Its limitations section is specific and candid.
- It openly corrects earlier published claims and values, all of which we verified independently.
- The public reproduction script regenerates every number and the figure exactly, which sets a high bar for computational work in the journal.
- The references are clean and genuine, and AI assistance is properly disclosed.

Priority Improvements Needed

Highest Priority: Report the census for seeds up to 200 in full. The paper presents it as a robustness check but gives only two of its categories. The omitted ones, 13.4% of seeds with no detected cycle and a further period-65 cycle, are directly relevant to the discussion of how common the cycles are and to the open problems you state. Adding them strengthens the paper rather than weakening it.

Secondary Priority: Position the work within the literature on cycle classification for Collatz-type maps. Single-amplification cycles are the natural analogue of Steiner's circuits, and the open multi-amplification problem is the analogue of the m -cycle problem studied by Simons and de Weger. Making this explicit clarifies what is new in your method and will

help readers from that community see the relevance of your results. Please also correct the cross-reference at p. 7, l. 277 and make the two citation corrections.

Additional Considerations: Each of the following would improve the paper further: a paragraph on why this recurrence matters beyond your earlier papers, a few clarifications of terminology, a sentence on how the non-cycling orbits grow, and archiving the code with a DOI.

Guidance for Revision Process

We recommend addressing the four Priority 1 items first. They are the required changes, and all of them are text-level. You can produce the extended census by extending your existing script so that it reports each distinct cycle found. The literature paragraph needs only the three references listed under Recommended Resources.

The Priority 2 and 3 items are recommended rather than required, and most take only a sentence or two each. If you use AI assistance, it is well suited to drafting the literature paragraph and to checking the revised census output against your script. Please verify every citation and number yourself, as you have done throughout this paper.

When resubmitting, please give the page and line of each change in a point-by-point response. This will allow a rapid re-review of the revised sections only.

Recommended Resources

- Böhm, C., & Sontacchi, G. (1978). On the existence of cycles of given length in integer sequences like $x_{n+1} = x_n/2$ if x_n even, and $x_{n+1} = 3x_n + 1$ otherwise. *Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur.*, 64, 260–264. Covers cycle equations for cycles with a prescribed pattern of steps.
- Steiner, R. P. (1978). A theorem on the Syracuse problem. *Proc. 7th Manitoba Conference on Numerical Mathematics* (1977), 553–559. The circuit theorem, which is the closest analogue of single-amplification cycles.
- Simons, J., & de Weger, B. (2005). Theoretical and computational bounds for m -cycles of the $3n+1$ problem. *Acta Arithmetica*, 117(1), 51–70. <https://doi.org/10.4064/aa117-1-3>. The analogue of cycles with several amplifications.
- Zenodo's GitHub integration (<https://zenodo.org>), for archiving a tagged release of the reproduction code with a citable DOI.

RESUBMISSION PROCESS

When resubmitting your revised manuscript, please provide:

1. Revised Manuscript (DOCX format) with updated version number

- All changes clearly incorporated
- Line numbers enabled
- Following AIR formatting guidelines

2. Cover Letter

- Brief summary of changes made
- Any significant decisions or departures from feedback

3. Point-by-Point Response Document

For each point raised in this evaluation:

- List the specific issue/suggestion
- Your response explaining how you addressed it
- Changes made, with specific page/line numbers in the revised manuscript
- If a suggestion is not followed, a clear explanation why

4. Submission Details

- Send as Reply to the same email address
- Subject line: RESUBMISSION - AIR-2026-000962-V2
- Include all three documents listed above as DOCX files

5. Resubmission Fees:

- First resubmission within 7 days: FREE
- After 7 days or 2nd+ resubmissions: \$10

CLOSING REMARKS

This paper resolves a precise question completely and correctly, discovers a new cycle that corrects the published record, and does so with unusual transparency and fully reproducible computations. These are exactly the qualities a strong mathematical paper should have.

The revisions requested are focused and additive. We are confident they are straightforward to make and will bring the paper to the acceptance threshold. We look forward to receiving the revised manuscript.

NEXT STEPS FOR AUTHORS**Revision Instructions****Immediate Actions:**

1. Review all feedback carefully
2. Develop a detailed revision plan
3. Address all critical issues (Priority 1)
4. Strongly consider all Priority 2 issues
5. Improve Priority 3 issues as feasible

When Ready to Resubmit:

1. Prepare the revised manuscript (DOCX)
2. Write a comprehensive point-by-point response
3. Send as Reply to the same email
4. Include Submission ID: AIR-2026-000962
5. Note the resubmission number and fee status

Timeline: Less than 1 week (AI-assisted) / 2–4 weeks (unassisted)

We wish you success in your research endeavors.

AIR Journals Editorial Team

<https://airjournals.org>