PAPER EVALUATION REPORT

AIR Journals — AI-Powered Academic Review

(Under Strict Scholarly Supervision)

Submission ID	***AIR-2026-001088***
Paper Title	Cycles with Exactly Two Amplifications in GCD-Augmented Fibonacci Recurrences
Author	Mosab Hawarey
Submission Date	11 November 2026
Journal	AIR Journal of Mathematics and Computational Sciences (DOI: 10.65737/AIRMCS) (ISSN: 3142-7197)
Review Completed	14 November 2026
Reviewer	AIR Journals AI Review System
Report Version	Final Evaluation Report

References are to sections, numbered results, tables and figures, which do not depend on pagination.

EXECUTIVE SUMMARY

Overall Score: 86/100

Editorial Decision: Minor Revisions

Decision Rationale: This is a correct, carefully scoped and independently reproducible paper. It answers the question left open in the author's previous paper by constructing explicit cycles with exactly two amplifications, the smallest of period 188, and it explains why they exist. We checked the main arguments and reproduced every computational claim independently, including all thirteen cycles of Table 1. Minor revisions are needed:

- give every long multiplier in Table 1 in full, and document the factorizations of L_q and how primality was proved;
- define two working terms, state the results imported from [5], and broaden the literature context;
- optionally, but highly recommended, reduce the self-citation in Section 7.

Primary Strengths:

- **Explicit, verified cycles.** Theorem 7 and Table 1 give thirteen cycles with exactly two amplifications, the smallest of period 188 with its state printed in full. All thirteen were confirmed by an implementation written from the manuscript text alone.
- **A clear mechanism.** For cycle multiplier L_q the determinant of the cycle equation in $\mathbb{Z}[\varphi]$ is a unit (Lemma 6), so every factorization of L_q is a candidate. Section 7 uses this to explain precisely why the expected analogy with the 2-cycles of the $3x + 1$ problem fails.
- **Reliable computations and honest scope.** The census of Theorem 11, the residue covering of Theorem 9 and the enumeration of Theorem 15 were reproduced exactly, and the scope labels S1–S4 state precisely what remains open.

Primary Concerns:

- **Incomplete Table 1.** Five of the thirteen long multipliers (rows 9–13) are given only by their number of digits, although the Abstract says that each cycle is "determined by three integers". The factorizations of L_q and the primality method behind Theorem 11 are not stated.

- **Self-citation in Section 7.** The sentence on in-context predictors cites four of the author's papers, [19]–[22], on in-context learning and GeoAI, and no independent work. Nine of the 22 references are the author's own. Revising this is highly recommended but optional, as it does not affect the integrity of the research.
- **Context and clarity.** The recurrence is not related to other gcd-perturbed recurrences, two working terms ("admissible", "local conditions") are not defined, and the proofs rely on results of [5] without restating them.

Timeline for Resubmission: Less than 1 week (if AI-assisted); 2–4 weeks otherwise

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COMPREHENSIVE EVALUATION RESULTS

Citation Integrity Verification (Step 1)

Status: PASS WITH WARNINGS

Verification Level: 100% — Every citation checked

Citation Statistics:

- Total bibliography entries: 22
- Bibliography entries cited in text: 22 (100%)
- Reference numbers cited in text that have bibliography entries: 22 of 22 (100%)
- References confirmed as genuine publications against publisher, journal, archive and database records: 22 of 22

Issues Identified:

Uncited Bibliography Entries (Padding): Count: 0 (0%) — Severity: None

Missing Bibliography Entries: Count: 0 — Severity: None

Additional Citation Issues:

- **Formatting consistency:** Good. Numbered citations in order of first appearance, with ranges and precise pinpoints such as [5, Proposition 13] and [3, Theorem 5 and Lemma 4], and APA-style entries used consistently. One small completeness item is listed below.
- **Self-citation rate:** 40.9% (9 of 22: [2]–[5], [18] and [19]–[22]). The predecessor papers [2]–[5] and the code archive [18] are justified and clearly disclosed ("The author's earlier work", Section 1). References [19]–[22], on in-context learning and GeoAI, are cited once, together, in one sentence of Section 7, with no independent source. This is the reason for the warning status.
- **Figure/table attribution:** Complete. Figure 1 and Table 1 present the author's own computations.
- **Data source citations:** Two small gaps. The software PARI/GP, named with its version in Section 6, is not cited, and the source of the factorizations of L_q used in the proof of Theorem 11 is not stated.

Critical (Must Fix):

1. Proof of Theorem 11: state whether the factorizations of L_q were computed for this work or taken from published factor tables, and cite the tables if so (see also Priority 1, item 1).

Important (Should Fix): None

Highly Recommended (Optional):

1. Section 7: revise the sentence on in-context predictors. Either remove it together with [19]–[22], which support only that sentence, or keep only the directly relevant reference and add at least one independent reference on in-context learning.

Removing [19]–[22] brings self-citation to 5 of 18 (27.8%). This change is optional, because it does not affect the integrity of the research itself.

Minor (Recommended):

1. [13]: add the journal DOI 10.4169/math.mag.87.5.323 alongside the arXiv link.
2. Section 6: cite PARI/GP 2.17.2.

Citation Integrity Conclusion: The bibliography is accurate and fully used: every entry is cited, every citation has an entry, and all 22 references are genuine. The warning concerns only the self-citation in Section 7, which is easily corrected; correcting it is highly recommended but optional. The explicit statement in Section 2 that the facts used from [2] and [3] are unaffected by their published corrections is good practice.

Plagiarism Content Assessment (Step 2)

Status: PASS

Tool Used: Premium Professional Service

Similarity Scores:

- Raw similarity score: 5%
- Adjusted similarity score (after removing legitimate matches): 0%
- Assessment: Green (0–14%)

Analysis: All 16 matches were classified as benign:

- 14 matches: bibliographic reference entries, i.e. titles, journal names and DOIs of properly cited works ([2]–[4], [9]–[11] and [18]–[22])
- 2 matches: boilerplate declarations (the AI-assistance disclosure, and the conflict-of-interest and funding statement)

Apart from these, there are no matches anywhere in the main text, including the results sections (Sections 4–6), Table 1 and Figure 1.

Self-Plagiarism Check:

- Author's prior work matched: Yes, the two declarations and some of the reference entries above, which also appear in one of the author's earlier articles on an unrelated subject
- Overlap: about 3% of the manuscript, all of it boilerplate or citation formatting
- Properly disclosed: Not required; no text from the methods, results or discussion is shared
- Properly self-cited: Yes; the related earlier papers [2]–[5] are cited precisely throughout
- Assessment: Acceptable

Required Corrections: None

Plagiarism Conclusion: No plagiarism detected. The text is original throughout, including where the paper builds directly on the author's earlier papers.

Quality Evaluation Results (Step 3)

Overall Quality Score: 86/100

Score Breakdown by Category:

CATEGORY A: CONCEPTUAL FOUNDATION

Score: 23/30

Evaluation:
The paper answers a precisely posed question. The author's previous paper [5] proved that exactly three cycles of the cofactor map have one amplification, and left two amplifications open as the analogue of the 2-cycles of the 3x + 1 problem, which do not exist (Section 1). The analogy is developed carefully, and the answer, that such cycles exist after all and why, gives the result interest beyond the recurrence itself (Section 7). The wider significance is less developed. The gap is documented only within the author's own series, and the paper does not place the recurrence among other gcd-perturbed recurrences studied elsewhere, such as Rowland's $a(n) = a(n - 1) + \gcd(n, a(n - 1))$. Doing so would show readers outside the series why the result matters.
Literature use is accurate and precise. The Collatz-cycle results are represented correctly (for example, no m-cycles for $m \leq 68$ [9] and $m \leq 91$ [10]), and every result imported from the author's earlier papers is cited to its numbered statement, such as [5, Proposition 13] and [3, Theorem 5 and Lemma 4]. Two points weaken the grounding. The sentence on in-context predictors in Section 7 rests only on the self-citations [19]–[22]. And the proofs rely heavily on [5], which is cited 19 times, without restating the results they use, so the paper cannot be checked on its own.
The exposition is precise. Step multiplier, amplification, cycle multiplier, content, pattern, segments and golden coordinate are all defined before use (Sections 2–3 and Definition 1), and the scope labels S1–S4 delimit every claim. Two points need attention:

- "Admissible q" (proof of Theorem 11; Section 7) and "local conditions" (Section 6; caption of Figure 1) are used without definition.
- The notation is dense, and in Lemma 10 the letter g, used elsewhere for a general step multiplier, is reused for L_q/h.

Key Strengths:

- A precisely posed open question, answered decisively and explained
- Accurate literature use, with imported results cited to their numbered statements

Areas for Improvement:

- Section 1: relate the recurrence to other gcd-perturbed recurrences (see Recommended Resources)
- Section 7 (optional, highly recommended): revise the sentence on in-context predictors (see Citation Integrity Verification)
- Section 2: state briefly the results imported from [5] (Proposition 13, Lemma 2 and Corollary 11(b))
- Define "admissible q" after Theorem 9 and "local conditions" at Lemma 4, add a short notation table, and rename L_q/h in Lemma 10

CATEGORY B: METHODOLOGICAL RIGOR

Score: 27/30
Evaluation:
The methods fit the questions exactly:

- Existence is settled by explicit construction (Theorem 7), the strongest possible form of evidence.
- Lemma 6 shows that $G = L_q$ is precisely the case in which the determinant of the cycle equation is a unit, which justifies concentrating the search there.

- Local obstructions at small primes (Lemma 4, Proposition 8) and a residue covering (Theorem 9) give the structure, and Lemma 13(c) turns the case $G > L_q$ into a finite search for each period.

Execution is excellent. We checked the main arguments, including Lemmas 1, 2, 6 and 10, Proposition 8, the case 6 | q of Theorem 9, Lemma 13(c) and Proposition 14, and found no error. We then re-implemented every computation from the manuscript text alone, without the author's code. The independent computation reproduced exactly:

- Theorem 7: the printed state for q = 188 and the factorization of L_{188}
- all thirteen rows of Table 1, including rows 9–13, which we recovered by factoring the primitive parts of L_{364}, L_{532} and L_{692}; and the properties stated in Proposition 8 and Corollary 12 for every printed multiplier
- Theorem 11 and Section 6: the 6,249,456 patterns with the failure breakdown 4,565,005 / 1,631,723 / 14,427 / 38,295, the six cycles with $q \le 360$, and the 97 local survivors with their exceptions at q = 10 and q = 28
- Theorem 9: the 31,080 residue classes and exactly the stated survivors
- Theorem 15: the 26 integral patterns for $q \le 36$, none of them a cycle, with about $1.6 \cdot 10^9$ congruences
- Proposition 14, the boundary recovery and the seed check of Section 6

For $360 < q \le 411$ we also ran a complete search over all factorizations of L_q (q = 398 aside, which Theorem 9 rules out) and found exactly the three cycles with q = 364.

Reproducibility is excellent. The code is archived publicly with a persistent identifier [18], every count is reported, and the computations could be reproduced from the text alone.

The remaining gaps are small:

- five long multipliers are not printed (Table 1, rows 9–13), so those cycles cannot be checked from the paper itself without factoring numbers of 61 to 144 digits or running the archived script;
- the factorizations of L_q, their source and the primality method are not given (proof of Theorem 11), and run times and hardware are not stated;
- a few steps are compressed to “one checks” or “direct” (Proposition 14; Lemma 13), and in Figure 1 the labels “q = 188” and “q = 236” overlap the axis tick labels and markers.

Key Strengths:

- Construction, structural proof and bounded exhaustive search, each matched to its question
- Correct arguments, and computations that we reproduced exactly from the text alone

Areas for Improvement:

- Table 1: print h and L_q/h in full for every row (an appendix is fine)
- Proof of Theorem 11: list or cite the factorizations of L_q, and state the primality-proving method
- Expand the “direct” cases of Lemma 13 and Proposition 14 in a line each, or point to the script function that checks them
- Section 7: add one or two sentences on what a proof for $G > L_q$ would require; Figure 1: move the overlapping labels

CATEGORY C: EVIDENCE & REASONING

Score: 18/20

Evaluation:

Every main claim is supported by proof, explicit construction or exhaustive computation:

- the existence of cycles with exactly two amplifications (Theorem 7)
- their structure (Proposition 8, Theorem 9, Lemma 10 and Corollary 12)
- the census (Theorem 11)
- the exclusion of short periods (Theorem 15)

The argument runs as a valid chain from Lemma 1 to Corollary 16. Hypotheses are stated where they are needed, such as a long segment of length at least 20 in Theorem 9, and $5s \le q$ and $3 \nmid q$ in Proposition 8(b), and they are checked when the results are applied (proof of Corollary 12). Conjecture 1 is clearly separated from the theorems, and its evidence is stated.

The few remaining points concern explicitness rather than validity:

- Corollary 12 uses only the exhaustive range $q \le 360$ of Theorem 11, which does not depend on Theorem 9. Saying so would make the absence of circularity immediately visible.
- The evidence for Conjecture 1 is modest (thirteen cycles at seven periods), and the search beyond $q = 360$ is complete for only 16 of the 30 admissible periods up to 716.

Key Strengths:

- Proof-level or exhaustive evidence for every theorem, with explicit, checkable witnesses
- A valid deductive chain, with the computer-assisted parts and the conjecture clearly identified

Areas for Improvement:

- Corollary 12: state that only the exhaustive range $q \le 360$ of Theorem 11 is used
- Section 7: report what is known at the 14 admissible periods in (360, 716] whose primitive parts are not fully factored

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CATEGORY D: TRANSPARENCY AND INTEGRITY

Score: 10/10

Evaluation:

Transparency is exemplary:

- The scope labels S1–S4 (Section 2) state exactly what is not proved, and Section 7 takes them up one by one as open problems, adding the unproved case $s = 1$.
- The computer-assisted parts are identified in the Abstract, in the list of contributions and in the statements of the theorems.
- The golden-coordinate criterion is presented as a restatement of [5], with no novelty claimed.
- The published corrections to [2] and [3] are addressed, and AI assistance is disclosed.

Assumptions are explicit throughout. The coprime-seed assumption A1 is justified by [2, Theorem 4], every imported fact is attributed to a numbered statement, the Fibonacci–Lucas identities are traced to Binet's formula, and the software and exact arithmetic are stated (Section 6). The only implicit item is the method used to prove the factors of L_q

prime, already noted above.

Key Strengths:

- Scope labels carried consistently from the Preliminaries to the Discussion
- Full traceability of assumptions and imported results

Areas for Improvement:

- Proof of Theorem 11: state the primality-proving method
- Optionally, add a sentence on how readers can check the computer-assisted parts independently

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CATEGORY E: CONTRIBUTION AND IMPACT

Score: 8/10

Evaluation:

The novelty is clear:

- the first known cycles of the cofactor map with exactly two amplifications, contradicting the expectation set by the 3x + 1 analogy (Section 1; Corollary 16)
- a new explanatory mechanism: the unit determinant at $G = L_q$ (Lemma 6; Section 7)
- a structure theorem (Theorem 9; Corollary 12) and a census that is complete up to q = 411 (Theorem 11)

The work is developed within the author's own framework, and its reach beyond this recurrence is modest. The contrast with the 3x + 1 problem is instructive, but the paper does not say whether the unit-determinant mechanism has analogues for other recurrences with gcd or division steps. Addressing this would widen the readership.

Key Strengths:

- A genuinely surprising result with a clear explanation

Areas for Improvement:

- Section 7: add 2–3 sentences on whether the mechanism transfers to related recurrences, and on what a proof for $G > L_q$ would need

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Summary of Score Breakdown by Category:

Category	*Score*	*Maximum*
Conceptual Foundation	23	30
Methodological Rigor	27	30
Evidence & Reasoning	18	20
Transparency and Integrity	10	10
Contribution and Impact	8	10
TOTAL	**86**	**100**

EDITORIAL DECISION

Decision: MINOR REVISIONS

Score Band: 86/100

Decision Thresholds	*Range*
Accept	90–100
Minor Revisions	80–89 ← This paper
Major Revisions	70–79
Major Revisions (Uncertain)	60–69
Reject (May Resubmit)	50–59
Reject	0–49

Factors Considered in Decision:

- Overall quality score: **86/100**
- Citation integrity status: PASS WITH WARNINGS (no impact on the decision; revising the self-citation is highly recommended but optional)
- Plagiarism assessment: PASS (no impact)
- Pattern: excellent rigor, evidence and transparency; narrower significance and reach
- Feasibility: all required changes are additions or corrections to the text; no new mathematics is needed
- Significance: the first cycles of this kind with exactly two amplifications, with a clear mechanism, in a specialized area

Detailed Justification

*This paper scores **86/100**, well within the Minor Revisions band.* The mathematics is correct: the arguments we checked hold, and every computational claim, from Table 1 to the census and the enumeration for $q \leq 36$, was reproduced independently. The score reflects the narrow reach of the problem and a few reporting gaps, not any defect in the results.

The required revision is specific: print every long multiplier of Table 1 in full, and document the factorizations of L_q and the primality method. Revising the sentence on in-context predictors in Section 7 and its four self-citations is highly recommended but optional. Neither change requires new mathematics or new computation: the numbers already exist in the author's outputs. The theorems, proofs and conclusions will remain unchanged.

Both integrity checks passed. All 22 references are cited and genuine, and the adjusted similarity is 0%. The citation check carries a warning for self-citation, which is concentrated in one sentence, does not affect the integrity of the research, and is easily fixed.

The contribution justifies publication: explicit cycles that overturn the expected analogy with the $3x + 1$ problem, a clear mechanism that explains them, and a rigorous structure theory and census. Its reach is currently specialized, and the recommended additions on related recurrences would broaden it. We expect the revised paper to meet the acceptance threshold.

Integration of All Evaluation Steps:

- **Step 1 (citation integrity):** a clean, fully verified bibliography, with one warning for self-citation in Section 7.
- **Step 2 (plagiarism):** original text; the only overlaps are reference entries and standard declarations.
- **Step 3 (quality evaluation):** excellent rigor, evidence and transparency. Improvements are needed in the completeness of Table 1, the documentation of the factorizations, the definition of two terms and the breadth of context.

Together, these support a Minor Revisions decision with high confidence of acceptance after revision.

REVISION REQUIREMENTS

Critical Issues (Must Address for Acceptance)

Priority 1 — Essential:

1. **Complete the Computational Record**
   - **Current Problem:** Table 1 gives the long multiplier h only by its number of digits for rows 9–13 (q = 364, 532 and 692), although the Abstract states that each cycle is "determined by three integers". The proof of Theorem 11 says that each L_q "was factored completely, with every factor proved prime", but the factorizations, their source and the primality method are not given.
   - **Required Fix:**
     - print h and L_q/h in full for all thirteen rows, in an appendix or a supplementary table if needed;
     - list the factorizations of L_q used in Theorem 11, or state their source and archive them with the code;
     - state how primality was proved (for example, the PARI/GP function and setting used).
   - **Success Criterion:** Each of the thirteen cycles can be reconstructed from the paper alone, and the source and certification of every factorization are stated.
   - **Importance:** The main result is an existence claim. Readers must be able to check every listed cycle, and the completeness claims rest on the factorizations.

Important Issues (Strongly Recommended)

Priority 2 — Significantly Improve Paper:

1. **Reduce the Self-Citation in Section 7 (Highly Recommended, Optional)**
   - **Current State:** The sentence "whether learned in-context predictors [19–22] can anticipate that prime from the pattern alone is an open question" cites four of the author's own papers, on in-context learning, geopotential fields and GeoAI, and no independent work. With them, 9 of the 22 references are the author's own (40.9%). This does not affect the integrity of the research, so the change is optional.
   - **Recommended Improvement:** Remove the sentence together with [19]–[22], or keep only the directly relevant reference and add at least one independent reference on in-context learning (see Recommended Resources). Removing [19]–[22] brings self-citation to 27.8%.
   - **Expected Benefit:** The only issue in an otherwise clean bibliography is removed, and the remark is supported by relevant sources.
2. **Define Working Terms and Simplify Notation**
   - **Current State:** "Admissible q" (proof of Theorem 11; Section 7) and "local conditions" (Section 6; caption of Figure 1) are not defined. The notation is dense, and g is reused for L_q/h in Lemma 10.
   - **Recommended Improvement:** Define both terms where they first appear, add a short notation table, and rename L_q/h in Lemma 10.
   - **Expected Benefit:** Sections 4–6 become easier to follow.
3. **Make the Paper Self-Contained with Respect to [5]**
   - **Current State:** The proofs use [5, Proposition 13] (the cycle equation, at most one cycle per pattern, and $G > \varphi^q$), [5, Lemma 2] and [5, Corollary

11(b)] without restating them.
   - **Recommended Improvement:** State these results briefly in Section 2.
   - **Expected Benefit:** The paper can be read and checked without [5].
4. **Expand Compressed Proof Steps**
   - **Current State:** "One checks" and "the cases m = 2, 4 are direct" (Proposition 14); "the cases q = 3, 4 are checked directly" (Lemma 13).
   - **Recommended Improvement:** Give a line for each, or point to the script function that performs the check.
   - **Expected Benefit:** Complete, checkable proofs.
5. **Broaden the Context**
   - **Current State:** The recurrence is not related to other gcd-perturbed recurrences, and wider implications and possible routes to the case $G > L_q$ (scope label S3) are not discussed.
   - **Recommended Improvement:** Cite one or two related recurrences in Section 1 (see Recommended Resources), and add 2–3 sentences in Section 7 on whether the unit-determinant mechanism has analogues, and on what a proof for $G > L_q$ would need (for example, a lower bound for $|G - \varphi^q|$).
   - **Expected Benefit:** Clearer significance and a wider readership.

Minor Issues (Suggested for Polish)

Priority 3 — Enhancement:

1. References: add the journal DOI 10.4169/math.mag.87.5.323 to [13], and cite PARI/GP 2.17.2 (Section 6).
2. Figure 1: move the labels "q = 188" and "q = 236" so that they do not overlap the axis tick labels or the markers.
3. Section 6: report run times and hardware for the searches behind Theorems 9, 11 and 15.
4. Corollary 12: state that only the exhaustive range $q \le 360$ of Theorem 11 is used.
5. Section 7: report what is known at the 14 admissible periods in (360, 716] whose primitive parts are not fully factored, as further evidence for Conjecture 1.

Revision Timeline and Expectations

Timeline: Less than 1 week (if AI-assisted); 2–4 weeks otherwise
Scope: Small, focused improvements to address specific issues
Re-review: Rapid review of revised sections only
Acceptance Probability: 95%+ if all feedback is addressed
Guidance: The changes are additions and corrections to the text; the theorems, proofs and computations will not need modification.

COMPREHENSIVE FEEDBACK FOR AUTHORS

What This Paper Does Well

This paper turns an expected non-existence result into an existence result, and explains why. The key observation is that for cycle multiplier L_q the determinant of the cycle equation in $\mathbb{Z}[\varphi]$ is a unit (Lemma 6), so every factorization of L_q yields an integral candidate and only the content conditions of Lemma 3 decide. From there, local obstructions at 2, 3, 5 and 7 (Lemma 4, Proposition 8) and a residue covering (Theorem 9) pin down the structure: $q \equiv 4$ or 20 (mod 24), a short segment of length 1, 2, 5 or 6, and a long multiplier built from primitive prime divisors of F_{2q}. The comparison with the 3x + 1

problem in Section 7, whose cycle denominator $2^K - 3^k$ equals 1 only for the trivial cycle, is short and illuminating.

The paper is also exemplary in its reliability and transparency:

- Every computational claim we tested was reproduced exactly by an implementation written from the text alone, from the period-188 state of Theorem 7 to the 6,249,456 patterns of Theorem 11 and the 26 candidates of Theorem 15.
- The scope labels S1–S4 state precisely what is proved, computed and conjectured, and Section 7 returns to each of them.
- The code is archived with a persistent identifier, every reference is genuine, the correction notices of the earlier papers are addressed, and AI assistance is properly disclosed.

Priority Improvements Needed

Highest Priority: Complete the computational record. Print every long multiplier of Table 1 in full, list or cite the factorizations of L_q behind Theorem 11, and state how primality was proved. With this, every cycle in the paper can be checked directly.

Secondary Priority: Define "admissible q" and "local conditions", state the results imported from [5] in Section 2, and expand the few proof steps written as "direct". These changes make the paper self-contained and easier to verify. We also highly recommend, although it is optional, revising the sentence on in-context predictors in Section 7: either remove it with [19]–[22], or support it with relevant independent work.

Additional Considerations: Relating the recurrence to other gcd-perturbed recurrences in Section 1 would broaden the paper's readership, as would a few sentences in Section 7 on whether the unit-determinant mechanism transfers and on what a proof for $G > L_q$ would need. The Priority 3 items (two reference details, the Figure 1 labels, run times, and the remark on Corollary 12) take a sentence each.

Guidance for Revision Process

We recommend addressing the Priority 1 item first. It is text-level: the missing multipliers and factorizations already exist in your computational outputs.

The Priority 2 and 3 items are recommended rather than required, and most take a sentence or two each; the optional change in Section 7 affects only one sentence and its references. If you use AI assistance, it is well suited to generating the supplementary table directly from your script's output and to cross-checking every number in the text against it. Please verify every citation and number yourself.

When resubmitting, please give the location of each change in a point-by-point response. This will allow a rapid re-review of the revised sections only.

Recommended Resources

- Rowland, E. S. (2008). A natural prime-generating recurrence. *Journal of Integer Sequences*, 11, Article 08.2.8. A well-known recurrence with a gcd step, useful as a comparison point in Section 1.
- Lagarias, J. C., Rains, E. M., & Sloane, N. J. A. (2002). The EKG sequence. *Experimental Mathematics*, 11(3), 437–446. Another gcd-defined sequence whose long-run behaviour is analysed rigorously.
- Garg, S., Tsipras, D., Liang, P., & Valiant, G. (2022). What can transformers learn in-context? A case study of simple function classes. *Advances in Neural Information Processing Systems*, 35 (NeurIPS 2022). https://arxiv.org/abs/2208.01066. A widely cited, independent reference for the in-context-learning remark in Section 7, if it is kept.

- Brillhart, J., Montgomery, P. L., & Silverman, R. D. (1988). Tables of Fibonacci and Lucas factorizations. *Mathematics of Computation*, 50(181), 251–260. https://doi.org/10.1090/S0025-5718-1988-0917832-6. The classical published tables, and a model for documenting factorizations and primality proofs.
- The PARI Group. *PARI/GP*, version 2.17.2. Université de Bordeaux. https://pari.math.u-bordeaux.fr/. Cite the version used; its documentation describes the primality tests behind isprime.

RESUBMISSION PROCESS

When resubmitting your revised manuscript, please provide:

1. Revised Manuscript (DOCX format) with updated version number
- All changes clearly incorporated
- Line numbers enabled | Following AIR formatting guidelines

2. Cover Letter
- Brief summary of changes made
- Any significant decisions or departures from feedback

3. Point-by-Point Response Document For each point raised in this evaluation:
- List the specific issue/suggestion | Your response explaining how you addressed it
- Changes made, with specific page/line numbers in the revised manuscript
- If a suggestion is not followed, a clear explanation why

4. Submission Details
- Send as Reply to the same email address
- Subject line: RESUBMISSION - AIR-2026-001088-V2
- Include all three documents listed above as DOCX files

CLOSING REMARKS

This paper resolves the question its predecessor left open, and in the unexpected direction: cycles with exactly two amplifications exist, and the reason is a clean algebraic one. The constructions are explicit, the structure theory is proved, and the computations could be reproduced exactly from the text alone. These are the qualities a strong paper in computational number theory should have.

The revisions requested are focused, mostly on the completeness of reporting and on context. We are confident they are straightforward to make and will bring the paper to the acceptance threshold. We look forward to receiving the revised manuscript.

NEXT STEPS FOR AUTHORS

Revision Instructions

Immediate Actions:

1. Review all feedback carefully and develop a detailed revision plan
2. Address all critical issues (Priority 1)
3. Strongly consider all Priority 2 issues and improve Priority 3 issues as feasible

Timeline: Less than 1 week (AI-assisted) / 2–4 weeks (unassisted)

We wish you success in your research endeavors.

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